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OPTIMAL SWITCHING CURVE FOR A MULTIPLE LINK WALKING ROBOT USING NELDER MEAD OPTIMIZATION

Paul Muench
Jamie MacLennan
US Army RDECOM / TARDEC
Warren, MI

James English, PhD
Energid Technologies
Boston, MA

ABSTRACT

Over the past several years, legged mechanisms have shown remarkable progress towards practicality. The need now is for efficient means of controlling these mechanisms. In previous papers, we have found the optimal switching curve for controlling a simplified model of walking called the rimless wheel model. In this paper, we again find the optimal switching curve for the rimless wheel using another approach--Nelder and Mead's simplex algorithm, a technique for global optimization. We compare the results from Nelder Mead with those obtained from our previous research using Pontryagin's Maximum Principle. We extend these results by applying Nelder Mead optimization to the walking gait of a multiple link model. We find an initial example of efficient control using this powerful technique.

INTRODUCTION

Legged systems have always held a promise (albeit a somewhat futuristic one) of mobility [1]. Until recently, most research in legged robotics has indeed focused on stability. Recent research [2] has come to the conclusion that stability is a necessary, but not sufficient condition. Efficiency is the key to gaining the true benefits of walking mechanisms. Particularly, we examine the efficiency of the powering scheme, *u(t),* of a legged robot. In previous papers [3-4] we have shown the existence of an optimal switching curve for a simplified model of walking based on Tad McGeer's rimless wheel model [5]. In this paper, we find an optimal switching curve for the rimless wheel model using Nelder Mead Optimization and compare the results from our previous analysis using Pontryagin's Maximum Principle. We extend these results to the walking gait of a multiple link model.

NELDER MEAD OPTIMIZATION

Nelder and Mead's simplex algorithm [6] is a popular, time-proven technique for optimizing general (nonsmooth) multivariable functions. Optimization algorithms of this type are called direct methods. They are robust and general and applicable to the problem of gait determination by parameterizing the gait and defining a metric over these parameters for optimization.

For function $f(\cdot)$ defined on an n-dimensional domain space, Nelder and Mead's method finds a minimum using a simplex formed with n+1 points. The process of finding a minimum proceeds through a process of transforming the simplex by 1) shifting points in the direction of a minimum, 2) shrinking, and 3) expanding. In addition, a restarting criterion can be used. This restarting criterion uses the concept of a simplex gradient. The Nelder-Mead algorithm continually updates a simplex in the N-dimensional space of the problem. For N=3, this simplex is a tetrahedron as illustrated figure 1. There are N+1 vertices and N+1 corresponding function values.

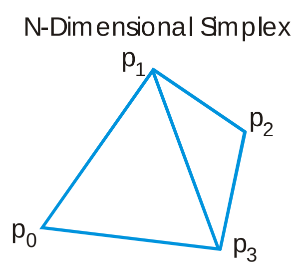

Figure 1: Nelder-Mead uses function evaluation over the vertices of an N-dimensional simplex to optimize a criterion function over an N-dimensional space. Shown here is a tetrahedron, which is a simplex for N=3. There are N+1 vertices in an N-dimensional simplex.

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Nelder Mead Steps

Order the points from lowest cost to greatest
Calculate the values of each simplex point
Try new point to replace the worst point. The algorithm attempts each step below in order until the cost of the proposed point is less than that of the worst point.

Reflection – reflects worst point about the center of the simplex

Expansion – expands in the direction of the best point

Contraction – shifts worst point towards the best point

Reduction – if no other options are available, *each* simplex point except the best is shrunk towards the center

Accept the point once a step successfully reduces the cost
Repeat process until a pre-specified tolerance is met

APPROXIMATION OF THE SWITCHING CURVE FOR THE SIMPLE PENDULUM

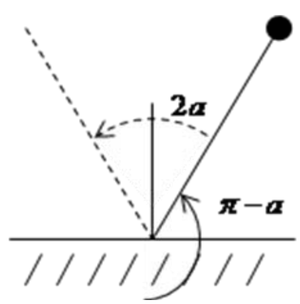


Figure 2: The rimless wheel model of walking acts like a powered simple pendulum (between foot collisions).

We can describe the cost of powering the rimless wheel as a combination of energy and time cost. We designate the cost, *J*, composed of a time cost, *T*, with a proportionality constant, *k* and an energy cost, *E* for a given function of torque, *u(t)*. The "optimal" control function *u*(t)* thus requires little energy but still reaches the destination quickly.

$$J = \int_{\pi-a}^{\pi+a} u d\theta + kT \qquad (1)$$

Equation 1 describes a *functional*, i.e., *J is* a function of a function. Since *u(t)* could take on any shape, we describe it using a high degree of freedom system (20-40 components). These functional components were optimized using Nelder-Mead in order to minimize the cost functional, *J*. As we saw in previous results using Pontryagin's Maximum Principle, when we optimized the torque function, we found evidence of a "switching curve." However, the result shown in figure 3 was only found after adjusting the initial simplex parameters and adding a sufficient number of components. We used Nelder-Mead to optimize the coefficients of a **Fourier series** (half-range expansion of the torque function) to approximate the solution.

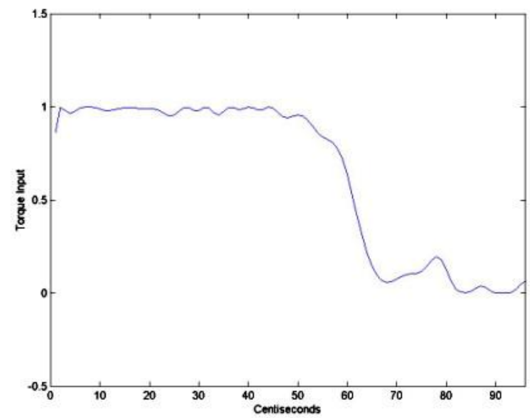


Figure 3: Fourier Series half range expansion found using Nelder-Mead Optimization

Likewise, we performed a Nelder-Mead optimization on the **cubic spline** components of the torque function. Pontryagin's Maximum Principle provides the correct answer to this problem at *k* = 5. The solution is a switching curve at 0.6s with a cost of 5.34. Nelder-Mead confirms this result.

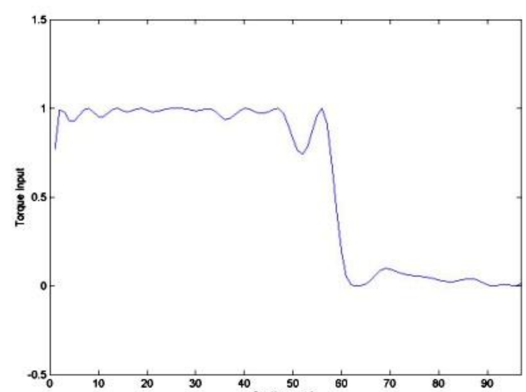


Figure 4: Spline Components found in Nelder-Mead Optimization

Note on Nelder Mead

Due to the many local minima, we found that it was critical to set the maximum value of torque, *u(t) < 1* for the simple pendulum. Without limiting the results to the maximum torque, Nelder-Mead would shoot briefly shoot above the maximum, in essence concluding that the square error we set was worth the penalty. To achieve a soft sort of clipping, we used a sine function to obtain the results shown. After Nelder Mead obtained a spline function for *u(t)*, we would feed Sin[*u(t)*] into the cost function. If we were to hard clip *u(t)* at the maximum, this might feed sharp corners into our Runge-Kutta integrator, like sticking a fork into the garbage disposal.

MULTIPLE LINK ROBOT CONTROL

To apply this approach to the gait optimization problem, a two-link pendulum model was constructed, as illustrated below. It has two rotational degrees of freedom.

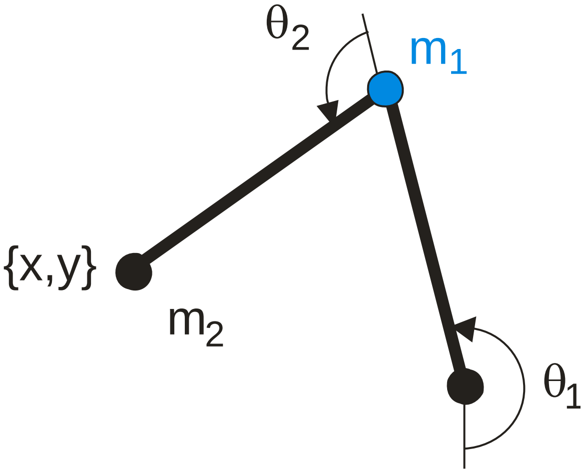


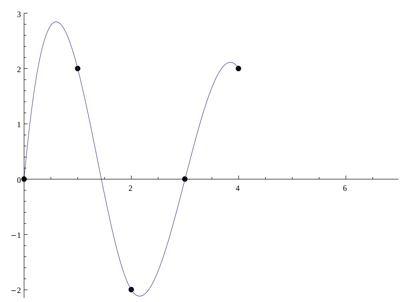
Figure 5: Multiple Link Robot Model used for illustrating optimized stride construction.

Each link length is 1 m. The masses of the links are at the ends of the links, and the mass values are variable—generally, $m_1 > m_2$. For optimizing the stride, the point in the lower right about which θ_1 rotates is assumed fixed. The location of the point at the end of the second link is represented in inertial coordinates as {x, **y**}. To parameterize a stride, the torque input on the second joint (θ_2 in the figure above) was constructed from a length-$\rho+3$ vector of real numbers:

$$\mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_\rho \\ x_{\rho+1} \\ x_{\rho+2} \\ x_{\rho+3} \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \vdots \\ \tau_\rho \\ T \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \qquad (2)$$

In (2), the last two numbers provide the initial conditions on the joint rates. The first ρ numbers are torques and form the domain values of knots for a spline, typically—but not necessarily—a cubic spline. Splines give a continuous and continuously differentiable function that is well behaved and practical to implement. These values are equally distributed over a time determined by the number in **X** at the $\rho+1$ position. Let this curve be defined as P[X]. The graphic below illustrates its formation.

$$\mathbf{X} = \begin{bmatrix} 0 \\ 2 \\ -2 \\ 0 \\ 2 \\ 4 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$



$$\mathbf{X} = \begin{bmatrix} 0 \\ 2 \\ 1 \\ 2 \\ 2 \\ 8 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

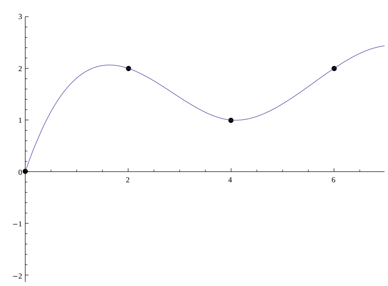


Figure 6: Illustrations of torque input functions P[**X**] constructed using the defined parameterization. The parameters $\dot{\theta}_1$ and $\dot{\theta}_2$ do not affect the shape of the curve.

Let $\boldsymbol{\tau}$ be the column vector of torques on the joints, $\boldsymbol{\theta}$ be the column vector of joint values (i.e., $\boldsymbol{\theta} = [\theta_1 \ \theta_2]^T$), $\mathbf{M}(\boldsymbol{\theta})$ be the inertia matrix, $\mathbf{C}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}})$ be the vector of Coriolis and centripetal terms, and $\mathbf{G}(\boldsymbol{\theta})$ be the vector of gravitational torques. With these definitions, the system dynamics for the model shown in 5 are given through the following:

$$\boldsymbol{\tau} = \mathbf{M}\ddot{\boldsymbol{\theta}} + \mathbf{C}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + \mathbf{G}(\boldsymbol{\theta}) \tag{3}$$

We do not consider frictional forces in this study. To form a metric for optimization, we set

$$\boldsymbol{\tau} = \begin{bmatrix} \mathbf{P}(\mathbf{X}) \\ 0 \end{bmatrix} \tag{4}$$

We integrate the differential equation in (3) over the time period of $\mathbf{P}(\mathbf{X})$ using the last two parameters in $\mathbf{X}$ as the initial joint rates. The initial joint positions are fixed in this study. A metric H for optimization is formed by combining several submetrics h_i using positive weights w_i as follows:

$$H = \sum_{i=1}^{N} w_i h_i \tag{5}$$

For our studies, the submetrics were selected from among the following:

The integral of the input torque squared:

$$h_i = \int_0^T \{P[x](t)\}^2 \, dt$$

The Euclidean error in end-point placement:

$$h_i = \sqrt{(x - x_d)^2 + (y - y_d)^2}$$

A measure of the difference in kinetic energy

$$h_i = \left(\dot{\theta}_i^T \mathbf{M} \dot{\theta}_i - \dot{\theta}_f^{\,T} \mathbf{M} \dot{\theta}_f\right)^2$$

With this choice of metrics, we were able to calculate strides by optimizing $\mathbf{X}$.

Example

For an example, the masses are set to $m_1 = 1/5$ and $m_2 = 1$. Gravity is 9.8 m/s^2 along vertical. The initial values for the joints are $\theta_1 = 5\pi/6$ radians and $\theta_2 = -2\pi/3$ radians. The metric is set to the following:

$$H = \int_0^T \{P[x]\}^2 \, dt + 8000\sqrt{(x - x_d)^2 + (y - y_d)^2} + \ldots$$
$$\ldots + 200\left(\dot{\theta}_i^T \mathbf{M} \dot{\theta}_i - \dot{\theta}_f^{\,T} \mathbf{M} \dot{\theta}_f\right)^2 \tag{6}$$

This is meant to be one example of a selection that could be used. With the high weights used, it effectively seeks to drive the point-placement error and the energy difference to zero while reducing the magnitude of the applied torque.

This configuration and metric selection was used with randomized Nelder-Mead optimization to calculate a stride by calculating 20 results and choosing the one with the smallest metric value. Based on using four torque values to build the spline curve, the following value was calculated:

$$\mathbf{X} = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \\ T \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} -0.931366 \\ -0.146139 \\ 0.245392 \\ -0.355268 \\ 1.07978 \\ 1.97858 \\ -1.10653 \end{bmatrix} \tag{7}$$

These give a metric value of 0.142565, and produce the following very low input torque:

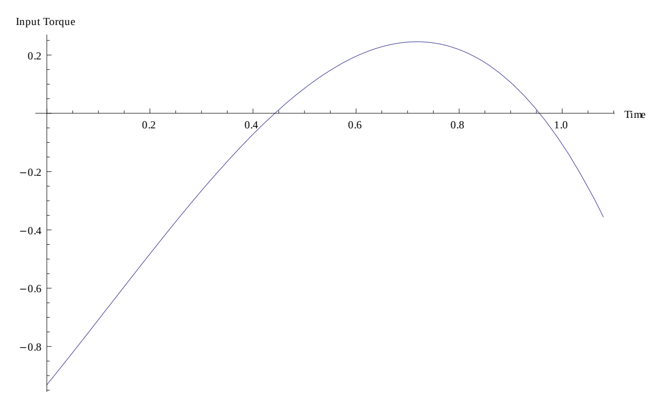


Figure 7: The input torque to drive the stride.

This torque, when input to the system defined through (3) and integrated gives the following output which is illustrated through a sequential rendering of the system in figure 8.

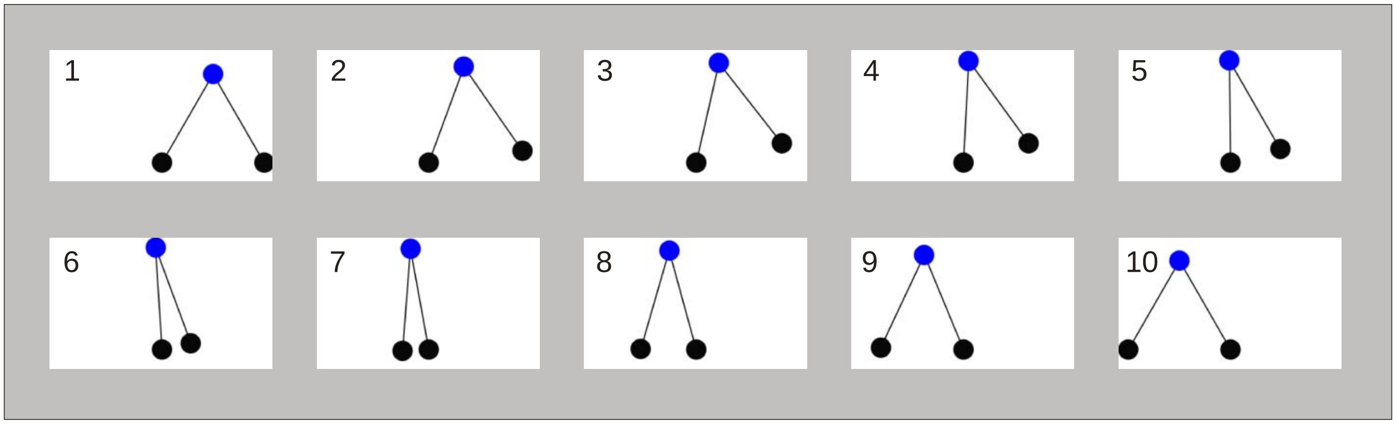


Figure 8: Screen shots from the optimized stride.

The stride produced by the input torque selected is shown in the figure above. The endpoint ends approximately symmetrically offset from its starting position and the system ends with approximately the kinetic energy with which it started.

CONCLUSIONS AND FUTURE WORK

The switching curve we have shown for the simple pendulum agrees closely with the global optimal control *u*(t)* obtained through Pontryagin's Maximum Principle (PMP) and other methods. The result we show for the double pendulum should be interpreted not as a claim of global optimal control, but as a typical example of an optimized stride for a more realistic walking model.

The results we found were not obtained by assuming any type of switching curve, i.e., we did not feed Nelder Mead the answer; however, we knew *a priori* that the result (at least for the simple pendulum) should look like a switching curve. Perhaps we found what we were looking for, because the results we show in figures 3 and 4 were not typical. In fact, you could find many local minima for *u(t)* that looked nothing like a switching curve but had a cost within ten percent or so of the optimal cost. Without the knowledge that a switching curve was the optimal control, we could have easily given up the search for the global minimum. Like traveling with a map, we found the previous results were a guide to Nelder Mead.

For the double pendulum we see an example of an optimized stride. Is this a global optimum? We simply do not know. We would like to further research these results using PMP or a similar technique. It may be that strict optimality is not critical, but rather knowing that you are close.

The nice thing about having a stationary switching curve in phase space is that you have a simple "map" to follow no matter where you find yourself. Thus, if you deviated from the optimal path, you could simply recalculate your position on the "map" and shoot for switching curve. Contrast that with trying to follow an optimal stride without this information. If you deviate too far from the optimal path, you might not be able to find your way back. That being said, for repetitive tasks like walking, it is likely desirable to strictly follow a known optimal stride.

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